# Distributed Problem-Solving In MAS: A Novel Generalized Particle Model

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Abstract. This paper presents a novel generalized particle model (GPM) for problem-solving in multi-agent systems (MAS) in complex environment. The complex environment involves multi-type coordination, multi-objective optimization, multi-degree autonomy, and multi-grain dynamics. The proposed GPM transforms a problem-solving process in MAS into the kinematics and dynamics of massive particles in a force-field. The GPA has many advantages in terms of the large-scale parallelism, the suitability for complex environment and the easier implementation with VLSI hardware technology.

Keywords Multi-agent systems, resource allocation, social coordination, social dynamics, generalized particle model.

### 1 Introduction

Most of approaches currently used to problem-solving in MAS have the following limitations and disadvantages:

- Usually only some simpler types of coordination in MAS, such as cooperation and competition, are taken into consideration. Furthermore, it is assumed that these kinds of coordination are always bilateral, aware, and conscious behavior.
- The MAS simply regards agents to be either completely selfish or completely unselfish.
- The global control, global information, and global objective are always required, so that it is difficult to realize in real-time the problem-solving in MAS.
- The influence of the availability of individual agents, such as congestion degree, failure status and priority level, is not well considered.
- Particularly, it is very difficult to treat the phenomena that randomly and emergently occur in MAS.

<sup>&</sup>lt;sup>1</sup> This work was supported by the National Natural Science Foundation of China under Grant No.60135010, No.60473044 and No.60073008, the National Key Foundational R&D Project (973) under Grant No.G1999032707, and the State Key Laboratory Foundation of Intelligence Technology and System, Tsinghua University.

To overcome the above-mentioned limitations, this paper proposes a novel generalized particle model (GPM), which transforms the problem-solving process in MAS into the kinematics and dynamics of massive particles in a force-field. The GPA has the following features:

- the ability to formalize typical types of social coordination among agents, including the unilateral, unaware and unconscious coordination;
- the ability to model different autonomy degree of individual agents with respect to the aggregate utility and personal utility, that is, the dual intentions for the whole systems and for agent itself;
- the higher parallelism to realize problem-solving in MAS under complex circumstances and with no global control and no global objective, and without using overall consistent information;
  - the ability to describe the the availability of individual agents;
- the openness to handle the events randomly and emergently occurred during MAS problem-solving;

#### 2 Generalized Particle Model for MAS

	$G_1$		$G_m$
$A_1$	$a_{11}(t), p_{11}(t), \zeta_{11}(t)$	•	$a_{1m}(t), p_{1m}(t), \zeta_{1m}(t)$
٠		•	•••
$A_i$	$a_{i1}(t), p_{i1}(t), \zeta_{i1}(t)$	$\cdot$	$a_{im}(t), p_{im}(t), \zeta_{im}(t)$
•			•••
$A_n$	$a_{n1}(t), p_{n1}(t), \zeta_{n1}(t)$	·	$a_{nm}(t), p_{nm}(t), \zeta_{nm}(t)$

Fig. 1. The assignment matrix  $S(t) = [s_{ik}(t)]_{n \times m}$  of task allocation and resource assignment in MAS, where  $s_{ij}(t) = \langle a_{ij}(t), p_{ij}(t), \zeta_{ij}(t) \rangle$ .

Consider the task allocation and resource assignment in MAS in complex environment. Given a finite set  $\mathcal{G}(\tau) = \{G_1, \dots, G_m\}$  of m task agents and a finite set  $\mathcal{A}(t) = \{A_1, \dots, A_n\}$  of n resource agents in the time session  $\tau$ , the resource agent  $A_i$  provides the task agent  $G_j$  with resource  $a_{ij}(t)$  at time t, and meanwhile the task agent  $G_j$  offers the payment  $p_{ij}(t)$  for unit resource of resource agent  $A_i$ . The resource agent  $A_i$  has the intention strength  $\zeta_{ij}(t)$  for task agent  $G_j$  through social coordinations among agents. We thus obtain an assignment matrix  $\mathcal{S}(t) = [s_{ik}(t)]_{n \times m}$ , as shown in Fig.1, where  $s_{ij}(t) = (a_{ij}(t), p_{ij}(t), \zeta_{ij}(t))$ .

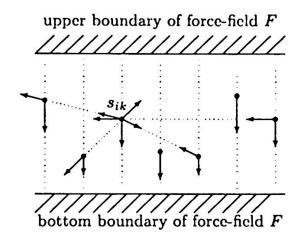


Fig. 2. Generalized particle model to optimize the task allocation and resource assignment in MAS.

A generalized particle model (GPM) to optimize the task allocation and resource assignment in MAS is shown in Fig.2, where the particle  $s_{ik}$  in a forcefield corresponds to the entry  $s_{ik}$  in assignment matrix S. A particle may be driven by several kinds of forces that are produced by the force-field, other particles and itself. The gravitational force produced by the force-field tries to drive a particle to move towards force-field boundaries, which embodies the tendency that a particle pursues maximizing the aggregate benefit of systems. The pushing or pulling forces produced by other particles are used to embody a variety of coordinations among agents. The self-driving force produced by a particle itself represents autonomy, personality and availability of individual agents in MAS. The larger the resultant force on a particle, the faster the motion of the particle. Based on a common dynamic equation, all the particles may move concurrently in a force-field. In this way, the GPM transforms the problemsolving in MAS into kinematics and dynamics of particles in a force-field. For simplicity and without loss of generality, we suppose that every particle may move along a vertical direction, that is, there is no horizontal component of forces on a particle. When all the particles reach their equilibrium states, we then accordingly obtain the solution to the optimization of task allocation and resource assignment in MAS.

Definition 1. Let  $u_{ik}(t)$  be the utility of particle  $s_{ik}$  at time t, and let J(t) be the aggregate utility of all particles. They are defined by

$$u_{ik}(t) = \alpha_1 \left[ 1 - \exp \left[ -(p_{ik}(t)p_k^*)(a_{ik}(t)a_i^*) \right] \right]; \tag{1}$$

$$J(t) = \alpha_2 \sum_{i=1}^{n} \sum_{k=1}^{m} u_{ik}(t)$$
 (2)

where  $0 < \alpha_2 < 1$ ,  $p_k^*$  and  $a_i^*$  are biases to embody the activity or availability of the task agent  $G_i$  and resource agent  $A_i$ , respectively.

Definition 2. At time t, the potential energy functions P(t) that is related to the gravitational force of force-field F is defined by

$$P(t) = \epsilon^2 \ln \sum_{i=1}^n \sum_{k=1}^m \exp[-u_{ik}^2(t)/2\epsilon^2] - \epsilon^2 \ln m \, n, \tag{3}$$

where  $0 < \epsilon < 1$ .

Definition 3. At time t, the potential energy function Q(t) that is related to interactive forces among particles is defined by

$$Q(t) = \xi \sum_{i=1}^{n} |\sum_{k=1}^{m} a_{ik}(t) - r_i(t)|^2 - \sum_{i,k} \int_0^{u_{ik}} \{ [1 + \exp(-\zeta_{ik}x)]^{-1} - 0.5 \} dx,$$
(4)

where  $0 < \xi < 1$ ; The  $r_i$  is the capacity of resource agent  $A_i$ . Note that the first term of Q(t) is for the capacity constraints of resource agents, that is realized through a special interactions among particles. The second term of Q(t) is caused by social coordinations among agents, where  $\zeta_{ik}$  is an intention strength.

Definition 4. The hybrid energy function of the particle  $s_{ik}$  at time t is defined by

$$\Gamma_{ik}(t) = -\lambda_{ik}^{(1)} u_{ik}(t) - \lambda_{ik}^{(2)} J(t) + \lambda_{ik}^{(3)} P(t) + \lambda_{ik}^{(4)} Q(t).$$
where  $0 < \lambda_{ik}^{(1)}, \lambda_{ik}^{(2)}, \lambda_{ik}^{(3)}, \lambda_{ik}^{(4)} \le 1$ . (5)

Definition 5. Suppose that the origin of coordinates is located on the central line between upper and bottom boundaries of force-field F. Let  $q_{ik}(t)$  be the current vertical coordinate of particle  $s_{ik}$  at time t. The dynamic equation for particle  $s_{ik}$  is defined by

$$\begin{cases} dq_{ik}(t)/dt = \Psi_{ik}^{(1)}(t) + \Psi_{ik}^{(2)}(t) & (6) \\ \Psi_{ik}^{(1)}(t) = -q_{ik}(t) + \gamma \, v_{ik}(t) & (6a) \\ \Psi_{ik}^{(2)}(t) = -w_{ik}u_{ik}(t) & (6b) \end{cases}$$

where  $\gamma > 1$ ;  $w_{ik} > 0$  is a positive weight coefficient; and the  $v_{ik}(t)$  is a piecewise linear function of  $q_{ik}(t)$  defined by

$$v_{ik}(t) = \begin{cases} 0 & \text{if } q_{ik}(t) < 0 \\ q_{ik}(t) & \text{if } 0 \le q_{ik}(t) \le 1 \\ 1 & \text{if } q_{ik}(t) > 1, \end{cases}$$
 (7)

Generalized Particle Model Algorithm (GPMA):

Costep 1. Initiate  $a_{ik}(t_0)$ ,  $p_{ik}(t_0)$  and  $q_{ik}(t_0)$  in parallel for  $i \in \{1, \dots, n\}, k \in \{1, \dots, m\}$ .

Costep 2. By the Eq.(1), calculate the utility  $u_{ik}(t)$  at time t in parallel for every particle  $s_{ik}$  in force-field F;

Costep 3. Calculate  $\Psi_{ik}^{(1)}(t)$  by Eq.(6a), and  $\Psi_{ik}^{(2)}(t)$  by Eq.(6b) in parallel for every particle  $s_{ik}$ . Then compute  $dq_{ik}(t)/dt$  in parallel for every particle  $s_{ik}$  by Eq.(6)

Costep 4. Calculate  $dp_{ik}(t)/dt$  and  $da_{ik}(t)/dt$  in parallel for every particle  $s_{ik}$  by the following Eqs.(8) and (9), respectively:

$$\frac{dp_{ik}(t)}{dt} = -\frac{d\Gamma_{ik}(t)}{dp_{ik}(t)} - \lambda_{ik}^{(5)} q_{ik}(t) 
= \lambda_{ik}^{(1)} \frac{\partial u_{ik}(t)}{\partial p_{ik}(t)} + \lambda_{ik}^{(2)} \frac{dJ(t)}{dp_{ik}(t)} - \lambda_{ik}^{(3)} \frac{dP(t)}{dp_{ik}(t)} - \lambda_{ik}^{(4)} \frac{dQ(t)}{dp_{ik}(t)} - \lambda_{ik}^{(5)} q_{ik}(t); (8) 
= \lambda_{ik}^{(1)} \frac{\partial u_{ik}(t)}{\partial dt} - \lambda_{ik}^{(5)} q_{ik}(t) 
= \lambda_{ik}^{(1)} \frac{\partial u_{ik}(t)}{\partial a_{ik}(t)} + \lambda_{ik}^{(2)} \frac{dJ(t)}{da_{ik}(t)} - \lambda_{ik}^{(3)} \frac{dP(t)}{da_{ik}(t)} - \lambda_{ik}^{(4)} \frac{dQ(t)}{da_{ik}(t)} - \lambda_{ik}^{(5)} q_{ik}(t); (9)$$
The second of  $\lambda_{ik}^{(5)} \leq 1$ 

where  $0 < \lambda_{ik}^{(5)} \le 1$ .

Costep 5. If  $dq_{ik}(t)/dt = 0$  and  $du_{ik}(t)/dt = 0$  hold for every particle  $s_{ik}$  at time t, then finish with success; Otherwise, using the obtained  $dq_{ik}(t)/dt$ ,  $dp_{ik}(t)/dt$  and  $da_{ik}(t)/dt$ , modify  $q_{ik}(t)$ ,  $p_{ik}(t)$  and  $a_{ik}(t)$  in parallel for every particle  $s_{ik}$  respectively by

$$q_{ik}(t + \Delta t) = q_{ik}(t) + \frac{dq_{ik}(t)}{dt} \Delta t,$$

$$p_{ik}(t + \Delta t) = p_{ik}(t) + \frac{dp_{ik}(t)}{dt} \Delta t,$$

$$a_{ik}(t + \Delta t) = a_{ik}(t) + \frac{da_{ik}(t)}{dt} \Delta t,$$
then go to Costep 2.

## 3 Properties of GPMA and Simulations

In this section, we gives the properties, including the correctness, convergency and stability of the GPM, and give some simulation results. For page limit, the proofs of all the theorems are omitted.

Theorem 1. Updating  $p_{ik}$  and  $a_{ik}$  by Eqs.(9), (10), respectively, gives rise to monotonically decreasing the hybrid energy function  $\Gamma_{ik}(t)$ , where very particle may autonomously determine its optimization objective according to its own personality and intention.

Theorem 2. The algorithm GPMA can dynamically optimize in parallel the task allocation and resource allocation in MAS in the context of multi-type coordination, multi-degree autonomy and multi-objective optimization for individual agents.

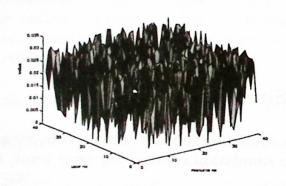
Theorem 3. If the following conditions:

$$\begin{cases} \gamma > 1 & (11a) \\ 1 - \gamma < w_{ik} \frac{du_{ik}(t)}{dq_{ik}(t)} \approx w_{ik} \frac{du_{ik}(t)/dt}{dq_{ik}(t)/dt} < 1. & (11b) \\ w_{ik}\alpha_1 < \gamma - 1 & (11c) \end{cases}$$
 remain valid, then equilibrium point of Eq.(6) is stable.

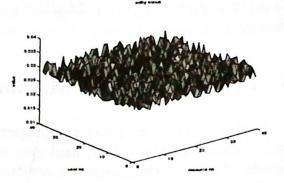
Theorem 4. If the conditions of the Eqs.(11a), (11b), (11c) remain valid, then every particle of the GPM will converge to a stable equilibrium coordinate position.

Some simulation results on the optimization of task allocation and resource assignment in MAS by using the algorithm GPAA are shown as follows.

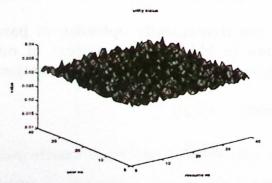
- The evolutionary process: The transient of personal utilities of all the particles during executing the algorithm GPMA is shown in Fig.3, which demonstrates every particle evolves simultaneously to its stable equilibrium state.
- The influence of problem size on utilities and performances: For the different problem size, the transients of the allocation fairness, aggregate resource utilization rate and aggregate users' satisfactory degree are shown in Fig.4.
- The comparisons: We further compare the algorithm GPMA with the famous MMA algorithm that is based on the Marketing Mechanism for resource assignment and task allocation in MAS. As shown in Fig.5, for different problem size the GPMA can all converge to a stable equilibrium solution much faster than the MMA. Moreover, the GPMA exhibits much better performance than the MMA in terms of the resource utilization rate and users' satisfactory degree, whereas they have almost approximately equal allocation fairness.



(a) Initial distribution



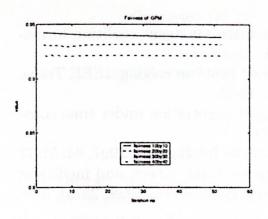
(b) Intermediate distribution

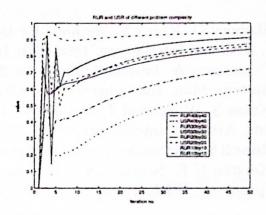


(c) Final stable distribution

The number of task agents:40; The number of resource agents: 40; The number of particles: 1600; Task agent demand:  $0.5 \sim 1.5$ ; Resource agent capacity:  $0.5 \sim 1.5$ ;  $\lambda_{ik}^{(1)} = 0.2$ ,  $\lambda_{ik}^{(2)} = 0.3$ ,  $\lambda_{ik}^{(3)} = 0.4$ ,  $\lambda_{ik}^{(4)} = 0.1$ ,  $\alpha_1 = 2$ ,  $\alpha_2 = 2$ ,  $\xi_2 = 0.1$ ,  $\zeta = 0$ ,  $\epsilon = 0.05$ .

Fig. 3. The utility distributions over all the particles at the beginning, intermediate and final stage of executing GPMA.





- (a) For allocation fairness
- (b) For resource utilization rate and aggregate users' satisfactory degree

Fig. 4. For different problem sizes, the transients of the allocation fairness, aggregate resource utilization rate and aggregate users' satisfactory degree during executing the algorithm GPMA.

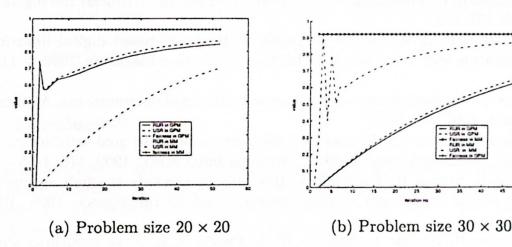


Fig. 5. For different problem sizes, the performance comparison between the GPMA and MMA in terms of transients of the allocation fairness, aggregate resource utilization rate and aggregate users' satisfactory degree.

### 4 Conclusions

We draw conclusions as follows:

- The proposed generalized particle approach can effectively solve the problems in MAS that involve multi-type social coordination, multi-degree autonomy and multi-objective optimization.
- The proposed generalized particle approach also has the advantages in terms of parallelism and feasibility for hardware implementation by VLSI technology.

### References

- [1] Shehory O, Kraus S. Methods for taks allocation via agent coalition formation, Artificial Intelligence, 1998, 101: 165-200
- [2] Chaudhury A. Two mechanisms for distributed problem solving, IEEE Trans. on System, Man, and cybernetics, 1998, 28(1): 48-55
- [3] Kraus S, Wilkenfeld J, Zlothkin G. Multiagent negotiation under time constraints, Artificial Intelligence, 1995, 75: 295-345
- [4] Russell S J. Rationality and intelligence, Artificial Intelligence, 1997, 94: 57-77
- [5] Kersten G E, Noronha S J. Rational agents, contract curves, and inefficient compromises, IEEE Trans. on System, Man, and cybernetics, 1998, 28(3): 326-338
- [6] Shoham Y, Tennenholtz M. On the emergence of social conventions: modeling, analysis and simulations, Artificial Intelligence, 1997, 94: 139-166
- [7] Pollock J L. The logical foundations of goal-regression planning in autonomous agents, Artificial Intelligence, 1998, 106: 267-334
- [8] Sandholm T W, Lesser V R. Coalitions among computationally bounded agents: Artificial Intelligence, 1997, 94: 99-137
- [9] Castelfranchi C. Modelling social action for AI agents, Artificial Intelligence, 1998, 103: 157-182
- [10] Weinstern P C, William P B, Durfce E H. Agent-based digital libraries: decentralization and coordination, IEEE Communication Magazine, 1999, 1: 110-115
- [11] Kraus S. Negotiation and cooperation in multi-agent environments, Artificial Intelligence, 1997, 94: 79-97
- [12] Shehory O, Kraus S, Yadgar O. Emergent cooperative goal-satisfaction in large-scale automated-agent systems, Artificial Intelligence, 1999, 110: 1-55
- [13] Kraus S, Sycara K, Evenchik A. Reaching agreements through argumentation: a logical model and implementation, Artificial Intelligence, 1998, 104: 1-69
- [14] Eriksson H, Shahar Y, Samson W T, Puerta A R. Task modelling with reusable problem-solving methods, Artificial Intelligence, 1995, 79: 293-326
- [15] Zaremba M B, Jedrzejek K J, Banaszak Z A. Design of steady-state behavior of concurrent repetitive processed: an algebraic approach, IEEE Trans. on System, Man, and cybernetics, 1998, 28(2): 199-212
- [16] Huang Z, Masuch M, ALX, an action logic for agents with bounded rationality, Artificial Intelligence, 1996, 82: 75-127